The valuation of chooser options for AAPL based on Monte-Carlo simulation

Yuchen Zhai[1, a, †] and Wenbin Zuo[2, b, †]

[1]Henan University of Economics and Law, Henan, 450000 China

[2]Rockford University, Rockford, IL 61108 USA

[a] nyuzyc@163.com, [b] wz176515@rockford.edu

[†]These authors contributed equally.

Keywords: Chooser option, volatility, interest rate, risk, stock price.

Abstract: With the development of economy, the financial market is becoming more and more complicated. As a result of high remuneration and deposits, most investors are seeking to find the optimal choice of investment to minimize risks and maximize benefits. Chooser options offer buyers considerable flexibility as they have the right to choose between call and put, becoming more and more popular. In this paper, we analyze AAPL's chooser options to find out factors that affect the value of chooser option based on Monte-Carlo Simulation. which is used to estimate the possible outcomes of an uncertain event and help to explain the impact of risk and uncertainty in prediction and forecasting models. According to the results, the higher the volatility of the underlying asset, the higher is the price for both call options and put options. In other words, when interest rates increase, the call option prices increase while the put option prices decrease. Whereas, if the choice time of contract is the current date the value of a chooser option is equal to the value of the simple call option. If the choice time is equal to one, the chooser value equals the value of a straddle strategy. However, we use the historical volatility, which is measured at a given time period, hence the results may not exactly predict what will happen in the future, which may also have an impact on the stock price. Most scientific studies have been focused on developing models for pricing various types of exotic options, but it is important to investigate their unique characteristics and to understand them correctly in order to use them proper market situations. These results shed light on investment decision making based on the elements that affect the price of chooser option and choices of buyers.

1. Introduction

The growth of financial markets and volatility of its participants encourage investors to look for the new, flexible financial instruments. The level of investment risk is increasing simultaneously, i.e., everyone operating in financial markets has to react suddenly to market changes and to correct his investment strategy in time. On this basis, investors are looking for new investment possibilities that could fit changing situation in the market and generate income from the investment. For the first time exchange listed options were traded in 1973. Since then, the volumes of their trade had risen sharply all over the world [1]. Option is a kind of financial instrument on account of the value of underlying securities. The option contract provides buyers the right to buy or sell an asset based on the categories of contracts. Each contract must have a particular expiration date for buyers to exercise option [2]. There are two kinds of investors: buyer and seller. Options bought by buyers are "Call options", while options sold by sellers are "Put options". Options that give investors flexibility to choose between call and put options are chooser option. Chooser option is a kind of European option with just one specific expiration date and strike price. Payoff of chooser options is based on the same method as vanilla. The distinction is that investors have the right to select the payoff they anticipate at the expiration date. If an underlying security is trading above its strike price at expiration, then the call option is exercised. The payoff is underlying price - strike price - premium or max(S-K,0) [3]. The investor benefits from buying the security at a lower price than it is selling for in the market. If a security is trading below its strike price at expiration, then the put option is exercised. The payoff is strike price – underlying price

- premium or max(K-S,0) [3]. Here, S is the stock price at expiration date, K is the strike price of option. The investor benefits from selling the underlying security at a higher price than it is trading for in the open market [4].

The one-month trend of AAPL stock has the shares in an extreme range from$145 in early August to $157 in early September. This price from AAPL shares implies that investors expect a positive earnings result. Contemporarily, option traders prefer call options than put options by nearly three to one, because the interest on call options is higher than on put options. Normally, this suggests that investors are expecting a positive earnings report, and traders appear to be expecting that AAPL will move higher after earnings. The call and put price are close with plenty of space to go downwards instead of upwards. This signifies that option buyers don't have a strong assurance about how the company will report, even though calls are being purchased over puts. Moreover, since there are amount of space in volatility range, it is difficult to predict the trend of share price. Anticipation of the sanguine earnings may put unexpected downward pressure on the share price of AAPL [5]. In order to find an alternative financial instrument to minimize the risk and maximize premium, we try to analyze the chooser option on AAPL, discussing elements and trend of chooser option with a clear graph to facilitate investors to determine whether they should choose call or put options. Based on the analysis, it is hoped to find out the optimal strategy for investors about how and when they should choose which kind of options, giving purchasers some pieces of advice.

The goal of research is to examine and analyze the valuation of chosen complex derivative security. Logical analysis and synthesis of scientific literature, comparative analysis and graphical modeling were used for the research.

2. Theoretical aspects of chooser option

A standard chooser option is purchased in the present while gives its holder the right to decide at some point in time, but whether the option will finally be a put or a call is unknown before maturity. Therefore, in this article the main attention is paid on calls and puts. Calls and puts are two type of stock options. A call option gives the holder the right to buy specified quantity of the stock at the strike price on or before expiration date. The writer of the option, however, has the obligation to sell the underlying asset if the buyer of the call option decides to exercise his right to buy. A put option gives the holder the right to sell specified quantity of the underlying stock at the strike price on or before an expiry date. The writer of the put option has the obligation to buy the agreed stock at the strike price if the buyer decides to exercise his right to sell. The option holder is the person who buys the right conveyed by the option. The option writer or seller is obliged to perform according to the terms of the option. Strike price or exercise price is the price at which the option holder has the right either to purchase or to sell the underlying asset [6].

A chooser option is suitable for investors who expect strong volatility of the underlying asset but who are not uncertain about direction of the change. When the underlying asset rises over the period of time, the holder of the option will choose the call option since it will have a higher value than the put option. In the case of falling value of the underlying asset, the choice will be the put option [7,8].

The most popular valuation model for options is the Black-Scholes model, which is based on the theory that markets are arbitrage free and assumes that the price of the underlying asset is characterized by Geometric Brownian motion. Another technique for pricing options is the binomial lattice model. In essence, it is a simplification of the Black-Scholes method as it considers the fluctuation of the price of the underlying asset in discrete time. This model is typically used to determine the price of European and American options [9]. The main options pricing models contain five factors that are used to determine a theoretical value for an option, and which have to be taken into account when pricing option contracts [10]:

market price of the underlying asset.
strike price.
time to expiration.
volatility of the underlying asset.

interest rates.

dividends expected during the life of the option.

As for market price and strike price, the payoff from a call option will be the amount by which the stock price in the market exceeds the strike price dealt with the option. Call options therefore become more valuable as the stock price increases and less valuable as the strike price increases. For a put option, the payoff on exercise is the amount by which the strike price exceeds the stock price [11]. Therefore, the put option becomes less valuable as the stock price increases and more valuable as the strike price increases.

Regarding to time to expiration, both put and call American options become more valuable as the time to expiration increases. European put and call options do not necessarily become more valuable as the time to expiration increases. This is because some owners of a long-life European option don't have the exercise opportunities open to the owner of a short-life European option.

As for volatility, it is a measure of how uncertain we are about future stock price movements. As volatility increases, the chance that the stock price will change in both directions increases. The value of both calls and puts therefore increases as volatility increases [10,12].

Regarding risk-free interest rate, it affects the price of an option in a less clear-cut way. Without additional assumptions, it is difficult to gauge the effect of increasing interest rates. Since increasing interest rates decrease the present value of the exercise price, there is a tendency for call values to increase and put values to decrease. It should be emphasized that these results assume that all variables remain fixed. In practice, when interest rates fall (rise), stock prices tend to rise (fall). The net effect of an interest rate change and the accompanying stock price change therefore may be different from that just given [6,10].

Dividends have the effect of reducing the stock price on the ex-dividend date. The values of call options are negatively related to the size of any anticipated dividend, and the value of a put option is positively related to the size of any anticipated dividend. Although Black-Scholes model is derived for valuing European call and put options on a non-dividend-paying stock, this model can be extended to deal with European call and put options on dividend-paying stocks, American options or options with different underlying assets [6,10-13]:

$$\mathrm{c} = Se^{-q(T-t)}N(d_1) - Xe^{-r(T-t)}N(d_2) \tag{1}$$

$$p = Xe^{-r(T-t)}N(-d_2) - Se^{-q(T-t)}N(-d_1) \tag{2}$$

where,

$$d_1 = \frac{In(S/X)+(R-Q+\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \tag{3}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{4}$$

Here, c is premium of European call option; p represents for premium of European put option; S is stock price; X is executed price; T-t is time to maturity; r denotes for risk free interest rate; q is dividends; σ is volatility of stock price and N1, N2 are the cumulative normal distribution function.

As mentioned before, a chooser option allows the investor to choose at a specific point in time t (t<T) whether the option is to be call or put. Once this choice was made at t, the option stays as either a call or a put to maturity (T) [14]. At this moment, the chooser has the same payoff as the straddle strategy, but it will be cheaper. The reason for the lower premium is that the straddle always has a payoff and the chooser can end in out-of-the-money.

An analytical solution for pricing the chooser option is possible because if the options underlying the chooser are both European and have the same strike price, put-call parity can be used to provide a valuation formula. This was proven by Rubinstein in 1991.

At time t the investor will choose the higher valued of the two options, i.e., one will choose the call if [14]:

$$C(S_t, K, T-t) > P(S_t, K, T-t) \tag{5}$$

According to the put-call parity $(\mathrm{P} + \mathrm{C} = \mathrm{C} + \mathrm{Ke}^{-\mathrm{r(T-t)}})$, then the equation reduces to:

$$C > C + \mathrm{Ke}^{-\mathrm{r(T-t)}} - S_t, \tag{6}$$

$$S_t > \mathrm{Ke}^{-\mathrm{r(T-t)}} \tag{7}$$

The investor will choose the call at t when the current stock price exceeds the present value of the strike price $S_t > \mathrm{Ke}^{-\mathrm{r(T-t)}} = K^*$. The payoff from the chooser at time T is presented in Table I [8,14,15].

At the time when the choice is made the value of the chooser option is [14,16]:

$$Chooser_{simple} = max[C(S_t, K, T-t), P(S_t, K, T-t)] \tag{8}$$

where C is the value of the call underlying the option and P is the value of the put underlying the option.

TABLE I: Payoff from the chooser options at time

Choice at time t	**Payoff at T**	
	$S_T \leq \mathrm{K}$	
Call	0	$S_T - \mathrm{K}$
Put	$\mathbf{K} - \boldsymbol{S_T}$	**0**

In the case of a simple chooser [14,16]:

$$P(S_t, K, T-t) = C(S_t, K, T-t) + \mathrm{Ke}^{-\mathrm{r(T-t)}} - S_t e^{-q(T-t)}. \tag{9}$$

Here, t follows that the payoff of a simple chooser is [14,16]:

$$C(S_t, K, T-t) + max\left\{0, \mathrm{Ke}^{-\mathrm{r(T-t)}} - S_t e^{-q(T-t)}\right\}, \tag{10}$$

and the current value of the option is

$$Chooser\ = e^{-q(T-t)} E[C(S_t, K, T-t) + \mathrm{e}^{-\mathrm{r(T-t)}} Emax\,\{0, \mathrm{Ke}^{-\mathrm{r(T-t)}} - S_t e^{-q(T-t)}\}]. \tag{11}$$

The expectations are taken at time t. The first term evaluates the current value of a call option with underlying asset price S, strike price K and time to maturity T-t, the second term is the value of a put with underlying asset price $Se^{-q(T-t)}$, strike price $\mathrm{Ke}^{-\mathrm{r(T-t)}}$ and time to expiration T-t [14,16]. Thus:

$$Chooser = C(S_t, K, T-t) + P\left(S_t e^{-q(T-t)}, \mathrm{Ke}^{-\mathrm{r(T-t)}}, T-t\right). \tag{12}$$

Based on Black-Scholes Eqs. (1) - (4) presented above, the value of a simple chooser option can be calculated [14]. A complex chooser option is similar to a standard chooser except that either the call/ put striking prices, call/put time to expiration or both are not identical. The payoff from a complex chooser is written as [10]:

$$Chooser\ max\,[C(S_t, K_t, T_t), P(S_t, K_t, T_t), t]. \tag{13}$$

3. Results & Discussion

At the time of the beginning, it is assumed that the strike price is \$140. There are two kinds of option including call option and put option. The price of call option after 1 year is 189\$ and the price of put option at that time is \$116. It is obvious that call option is a better choice as $max[C(S_t, K, T-t), P(S_t, K, T-t)]$ here equals to \$49, which is the benefit of call option at time 1 year. Therefore, buyers may choose the call option rather than put option. Nevertheless, this is not the final payoff. Investors just choose which option they want to buy at time 1 year and they cannot change their option from time 1 year to the exercise date. At time 2, the price of call option is \$197 while that of the put option is \$118. Since investors have already chosen call option at time 1, they have to exercise their right at the price of call option at time 2. The ultimate payoff of this chooser option is the difference between $197 and $140 which is the strike price.

The choice of chooser option is presented in Fig. 1, where blue line is the call option and red line is the put option.

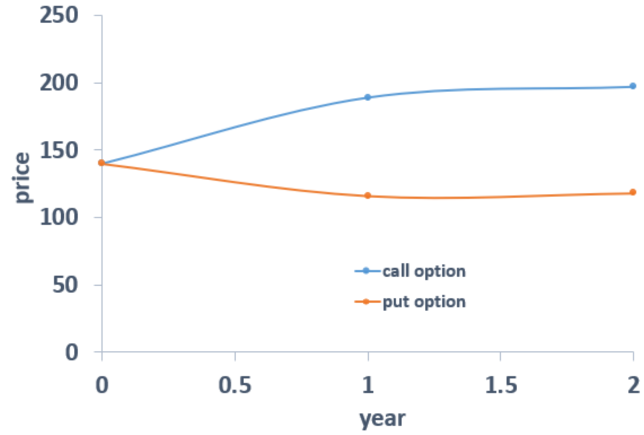

Fig. 1. The price of call and put option

Volatility is the implication of the fluctuation in the market and is the measurement of speed and amount of movement for underlying asset prices. It is found out that when the historical volatility increase, the prices of call and put option will all go higher. As the volitivity varies, the price of call option and put option of the simple chooser option change too and are as presented in Table II.

Table II: Relationship between volatility and price of call option and put option

Volatility (%)	Price of Call Option ($)	Price of Put Option ($)
15	178	143
23	207	159

The effect is inverse when the volatility goes down. The rationale is that high volatility means high risk in the market. When the volatility increases, implicating that the risk will also go upward, the buyers of call option will forego premium and buyers of put option will take more profit. When volatility go down, it means that risk of market reduces. Therefore, more people buying call options will take profits. That's the reason why high volatility makes options more valuable. People prefer to choose long put options with high volatility and short call option with low volatility.

The data and results are presented at Figs. 2 and 3, where blue lines denote for the call option and red lines represent the put option.

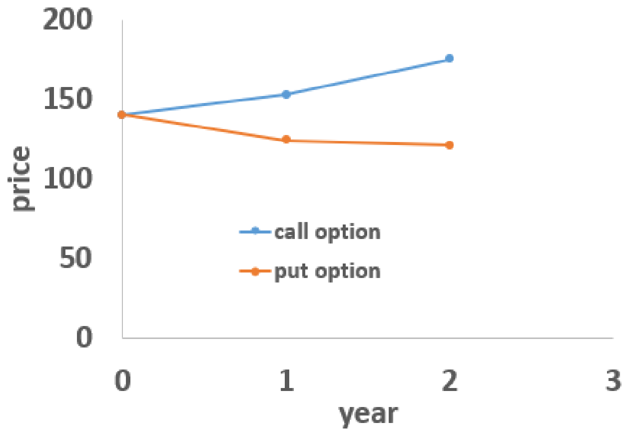

Fig. 2. The relationship between volitivity in 15% and chooser price.

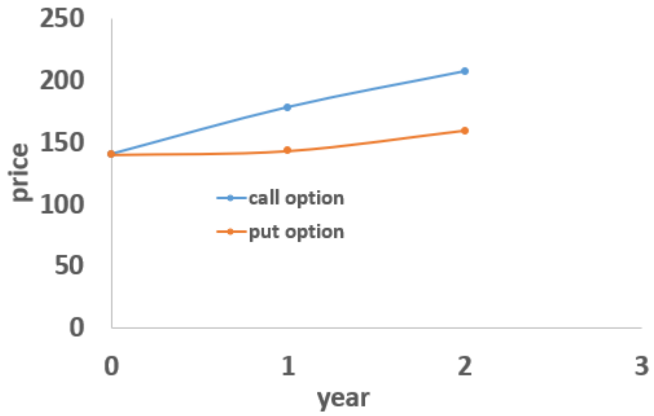

Fig. 3. The relationship between volitivity in 23% and chooser price.

After meticulous research and analyzing, we find out a couple of results from these data.

The main factors in option pricing are market price of the underlying asset, strike price, volatility of the asset, time to maturity of the contract, interest rates and dividends. The same factors affect prices of exotic options, too.

The higher the volatility of the underlying asset, the higher is the price for both call options and put options. The upside helps calls and downside helps put options.

Interest rate: When interest rates increase, the call option prices increase while the put option prices decrease.

If the choice time of contract is the current date the value of a chooser option is equal to the value of the simple call option. If the choice time is equal to one, the chooser value equals the value of a straddle strategy.

Correlation between the chooser value and strike price is not strong but the influence of time of choice must be taken into consideration because of its influence on chooser option price.

4. Conclusion

In general, we investigate 1000 samples in Monte-Carlo Simulation and evaluate value of chooser options for AAPL in Black-Scholes model. Specifically, we assume the strike price of the option and the expiration rate as well as use the stock simulation formula in Excel to calculate payout of each sample, taking into account the volatility, dividend, interest rate and stock price of AAPL. The relationships of chooser option based on the strike price and automatic choice of options are presented. According to the analysis, the higher the volatility of the underlying asset, the higher is the price for both call options and put options. The upside helps calls and downside helps put options. When interest rates increase, the call option prices increase while the put option prices decrease. If the choice time of contract is the current date the value of a chooser option is equal to the value of the simple call option. If the choice time is equal to one, the chooser value equals the value of a straddle strategy. Nevertheless, due to the usage of historical volatility, results may not exactly predict what will happen in the future, and they may also have an impact on the stock price. These can be improved based on applying least square analysis to the research to analyze the entire simulated path instead of the terminal value of the path, providing more accurate results.

Moreover，Financial instruments traded in the markets and investors situation in such markets are getting more and more complex, which leads to more complex derivative structures used for hedging that are harder to analyze and which risk is harder managed. On account of the complexity of these instruments, the basic characteristics of many exotic options may remain unclear. Most scientific studies have been focused on developing models for pricing various types of exotic options, but it is important to study their unique characteristics and to understand them correctly in order to use them in proper market situations. These results offer a guideline for choose option pricing.

5. Conflict of Interest

The authors declare no conflict of interest.

6. Author Contributions

These authors contributed equally.

References

[1] Raimonda Martinkutė-Kaulienė EXOTIC OPTIONS: A CHOOSER OPTION AND ITS PRICING

[2] Chen, J. 2021, September 27. How options work for buyers and sellers. Retrieved October 11, 2021, from https://www.investopedia.com/terms/o/option.asp

[3] Wikipedia contributors. (2019, June 14). Chooser option. In Wikipedia, The Free Encyclopedia. Retrieved October 11, 2021, from https://en.wikipedia.org/w/index.php?title=Chooser_option&oldid=901762277

[4] Mitchell, C. 2021, September 13. What is a chooser option? Retrieved October 11, 2021, from https://www.investopedia.com/terms/c/chooseroption.asp

[5] Scott, G. 2021, July 26. Apple (AAPL) option traders enthusiastic ahead of earnings. Retrieved October 11, 2021, from https://www.investopedia.com/aapl-option-traders-enthusiastic-ahead-of-earnings-5194191

[6] Jarrow, R. A.; Rudd, A. 1983. Option pricing. USA: Richard D. Irwin, INC. 235 p.

[7] Simons, H. L. 2000. Choose your friends, choose your options, Futures: News, Analysis & Strategies for Futures, Options & Derivatives Traders 29(5): 56–58.

[8] Deacon, C.; Faseruk, A.; Strong, R. 2004. Suggested refinements to courses on derivatives: presentation of valuation equations, pay off diagrams and managerial application for second generation options, Journal of Financial Management & Analysis 17(1): 62–76.

[9] Bampou, D. 2008. Numerical methods for pricing exotic options: Master thesis. Available from Internet: http://www3.imperial.ac.uk/pls/portallive/docs/1/45415707.PDF.

[10] Hull, J. 2000. Options, Futures, and Other Derivative Securities. Fourth Edition. New Jersey: Prentice Hall. 698 p

[11] Laurence, P.; Avellaneda, M. 2000. Quantitative modeling of derivative securities from theory to practice. Chapman&Hall/CRC. 322 p.

[12] Martin, J. S. 2001. Applied math for derivatives: a non-quant guide to the valuation and modeling of financial derivatives. Asia: John Wiley & Sons. 447 p.

[13] Whaley, R. E. 2006. Derivatives: Markets, Valuation and Risk Management. New Jersey: John Wiley & Sons. 930 p.

[14] Cuthbertson, K.; Nitzsche, D. 2003. Financial engineering Derivatives and risk management. John Wiley and Sons, LTD. 776 p.

[15] Sandmann, K.; Wittke, M. 2010. It’s your choice: a unified approach to chooser options, International Journal of Theoretical & Applied Finance 13(1): 139–161. http://dx.doi.org/10.1142/S0219024910005711

[16] Rubinstein, M.; Reiner, E. 1992. Exotic options. Working Paper 220. Available from Internet: http://www.haas.berkeley.edu/groups/finance/WP/rpf220.pdf.